Addendum: Some explanatory notes

The role of the generating plane, already mentioned in the text of the work, is discussed below in greater details.

The plane is characterized by a fluid and by a solid particle bearing the same density.

Denoting by ρ_s the density of the solid particle and by ρ that of the fluid, further denoting by $(\pm)$ μ an excess or a defect in density, one writes for the particle and the fluid in the generating plane:

$$\frac{\rho_s}{\rho} = (1 \pm \mu) \to 1 \qquad (1)$$

In Eq. (1) one associates to $+\mu$ the properties of a setting particle, and to $-\mu$ those of a buoyant one, with condition (1) referred to an "indifferent particle" (versus the force due to gravity).

Denoting by Ω_{10} the stream velocity, by Δ_{10}^{0} the pipe diameter and by x the volumetric concentration of the solid, one will write for one particle only present in the system:

$$x \to 0 \qquad (2)$$

Moreover in the generating plane the particle considered will be of a diameter δ_{10} of the same size of the pipe diameter, i.e. one will write:

$$\delta_{10} = \Delta_{10}^{0} \qquad (3)$$

A Cartesian system of axes is drawn in the plane, where on the axis of the abscissae the velocity Ω_{10} is represented and on the axis of the ordinates, the pipe diameter Δ_{10}^{0}.

Then one traces on the plane a parabola of equation as below:

$$\Delta_{10}^{0} = \Omega_{10}^{2} \qquad (4)$$

One selects on this locus two values $\bar{\Delta}_{10}^{0}$ and $\bar{\Omega}_{10}$ such that

$$\bar{\Delta}_{10} = \bar{\Omega}_{10}^{2} \text{ with } \bar{\delta}_{10} = \bar{\Delta}_{10}^{0} \qquad (5)\ \{3,12\}$$

These values define on the parabola (4) a point $\bar{P}$.

Let one draw per point $\bar{P}$ the hyperbola:

$$\Omega_{10} \cdot \Delta_{10}^{0} = \overline{RE}_{10} = \text{constant} \qquad (6)$$

where by $\overline{RE}_{10}$ one denotes the Reynolds number of the stream

$$\overline{RE}_{10} = \bar{\Delta}_{10}^{0} \cdot \bar{\Omega}_{10} \qquad (7)$$

Then one can simply state the generating plane is now fully defined graphically by a parabola and a hyperbola intersecting at the point $\bar{P}$.

The transport af an "indifferent particle" inside the pipe is assumed to take place herein without any loss of energy.

This is a "virtual condition" associated with any point of the generating plane.

Considering now a density variation $(\pm)\mu$ enforced on the particle at $\bar{P}$, then one leaves the plane and enters the space with $(+)\mu$ above and (-)μ below the plane, respectively.

Then, one can imagine to reach e.g. point Q_+ from $\bar{P}$, if a suitable function g(f) of the friction factor f can be defined for the space above the plane and the same for the point Q_- in the space below.

But this function already exists and can be recovered from the text of the work.

It is repeated below in a more general form and associated once with the transport of a spherical particle and once with the transport of a cylindrical one.

$$\frac{1}{2}\ \frac{\mp f_{10}}{\mp \mu_{10}} = \begin{cases} 0{,}0607 & sphere \\ 0{,}0405 & cylinder \end{cases} \qquad (8)^{\{3\ .\ 13\}}_{\{6\ .\ 5\}}$$

In (8) to the (-) sign of the denominator there corresponds a (-) sign of the numerator, i.e. $-f_{10}$, in agreement with a convention already introduced in worked example 3 of the text.

Let one discuss the meaning of Eq. (8) more closely.

The "virtual indifferent particle" of the generating plane has become now a real one, being outside the generating plane.

For this a finite positive (negative) friction factor f for a positive (negative) density excess μ is required.

Hence the experimental hydraulics will ultimately give an answer to this problem by a suitable value of f_{10}.

The friction factor f_{10} of a smooth wall can be easily determined from the value $\overline{RE}_{10}$ as in (7).

From the above premises one can derive an easy interpretation of Fig. 1.

Considering the AB locus at point A of ordinate 0,0607 and abscissa 1, one deals with a particle of size $\delta_{10} = \Delta_{10}^{0}$ in presence of a finite friction factor f_{10} and an excess density μ_{10}.

Let one proceed along AB with μ increasing and δ decreasing as below:

$$\delta \cdot \mu = \Delta_{10}^{0} \cdot \mu_{10} = constant \qquad (9)$$

Upon reaching B, the particle diameter δ has become with μ:

$$\delta_B = \frac{\Delta_{10}^{0}}{11455} \text{ and } \mu_B = \mu_{10} \cdot 11455 \qquad (10)$$

Passing now to Fig. 2, the condition "one particle only" is rescinded.

One might consider many small particles now present in the system, (tentatively) up to a solid concentration $x = 0{,}40$, which still insures no mechanical contact between particles.

With reference to Fg. 3 (insert) one has represented a cylindrical particle consisting of four elementary cylinders, each containing one particle. Upon increasing the particle diameter, this and the enclosing elementary cylinder will grow to the size of the pipe, i.e.:

$$\delta \rightarrow \Delta_{10}^{0} \tag{11}$$

Let one introduce a linear concentration *x* as below:

$$x = \frac{\delta}{\Delta_{10}^{0}} \tag{12}$$

Then in Fig. 3:

a.

one plots for the elementary cylinder the values:

$$x^{1/2} = \left(\frac{\delta}{\Delta_{10}^{0}}\right)^{\frac{1}{2}} \ versus \ x = \frac{\delta}{\Delta_{10}^{0}} \ with \ 0 < \frac{\delta}{\Delta_{10}^{0}} < 1 \text{, i.e. inside the range (0-1)} \tag{13.a}$$

b.

one plots for the spherical particle contained in the elementary cylinder the values:

$$x^{1/2} = \left(\frac{\delta}{\Delta_{10}^{0}}\right)^{\frac{1}{2}} \ versus \ x = \frac{\delta}{\Delta_{10}^{0}} \ with \ 0 < \frac{\delta}{\Delta_{10}^{0}} < 0{,}8165 = \left(\frac{2}{3}\right)^{\frac{1}{2}} \text{, i.e. inside the range (0-0,8165)} \tag{14.b}$$

Then conditions (13.a) and (14.b) define two parabolas (a) and (b) for the elementary cylinder and contained particle, respectively.

One can pass from parabola (a) to parabola (b) using a multiplier

$\beta = 1{,}11 = \frac{1}{0{,}90}$ i.e. writing:

$$x_b^{\frac{1}{2}} = 1{,}11 \ \ x_a^{\frac{1}{2}} \tag{15}$$